ADComms Final Project: Hardware Radio Simulation

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1 Introduction

Amplitude modulation (AM) is a common method for encoding message signals onto a known carrier wave. This signal can then be received and demodulated using a rectifier, low pass filter, and multiplier given a known carrier frequency [1]. AM is most commonly used in car radios. For this project, the team intended to simulate their own custom radio.

The team's objective was to translate their theoretical learnings about radio to the real world. They wanted to develop their own understanding of the various components within a signal processing communications system.

The team wanted to delve into signal generation, transmitter, and receiver circuits. From their research, the team created their own versions of these circuits in the simulation software LTspice. The team was able to test these simulated circuits and compare with their knowledge learned in the Analog & Digital Communications Course to check their results.

2 System Diagram

At the highest level, there are two primary components needed for AM modulation–a transmitter and receiver. Figure 1 shows the system diagram of the AM radio that was implemented. The transmitter encodes a message onto a carrier signal of the desired frequency. For the case of AM, the encoding occurs by scaling the amplitude based on the input signal, thus encoding the signal onto the envelope of the signal. This allows for the signal to be sent at a higher frequency. The modulated signal is then received by a receiver after passing through a channel, which can be anything from air to a wire. For this particular implementation, the channel was a simple wire without any loss or added noise. The received signal is then be demodulated using an envelope detector which first rectifies the signal and removes the high frequency carrier wave.

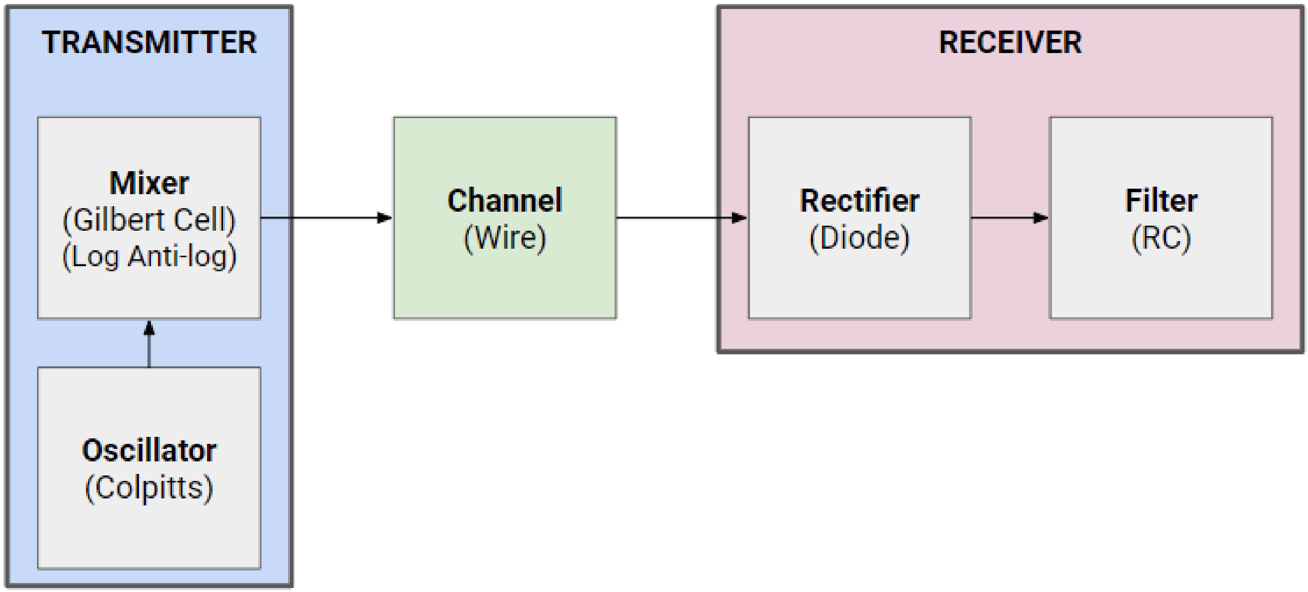

Figure 1: System diagram of the simulated AM radio

3 Transmitter

The transmitter is built using the components discussed earlier – an oscillator and multiplier. The output of the oscillator creates a carrier wave to be modulated with a message signal. To get the modulation to occur, namely amplitude modulation, both the carrier and message must be passed into a multiplier. This results in a modulated signal which can be sent over a desired frequency to be received by a receiver, which will demodulate the signal.

3.1 Oscillator

The Colpitts oscillator was invented by Edwin H. Colpitts in 1918 [3]. Part of the LC oscillator family, this oscillators uses an inductor (L) and a capacitor (C) of a particular values to generate oscillations at a fixed frequency.

3.1.1 Tank Circuit

The fundamental unit of this LC oscillator is a tank circuit. A schematic for a tank circuit is shown in Figure 2.

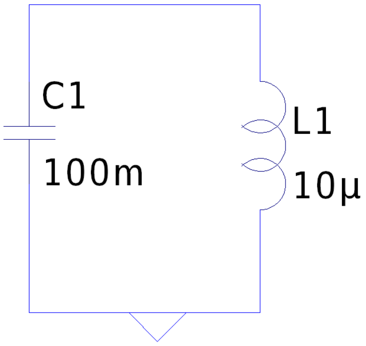

Figure 2: LTSpice Schematic of a Tank Circuit

A tank circuit operates by converting energy between different storage forms. Initially, the capacitor is charged up and the connected to an inductor as shown in Figure 2. The electrostatic energy stored in the capacitor is discharged over time generating a current flow. This results in the energy being stored in magnetic fields in the inductor. Once the capacitor is depleted, the inductor, which resists changes in current, begins to charge the capacitor in the opposite direction. This process repeats and generates a sinusoid. With ideal components the plot would look like the result in Figure 3 [9].

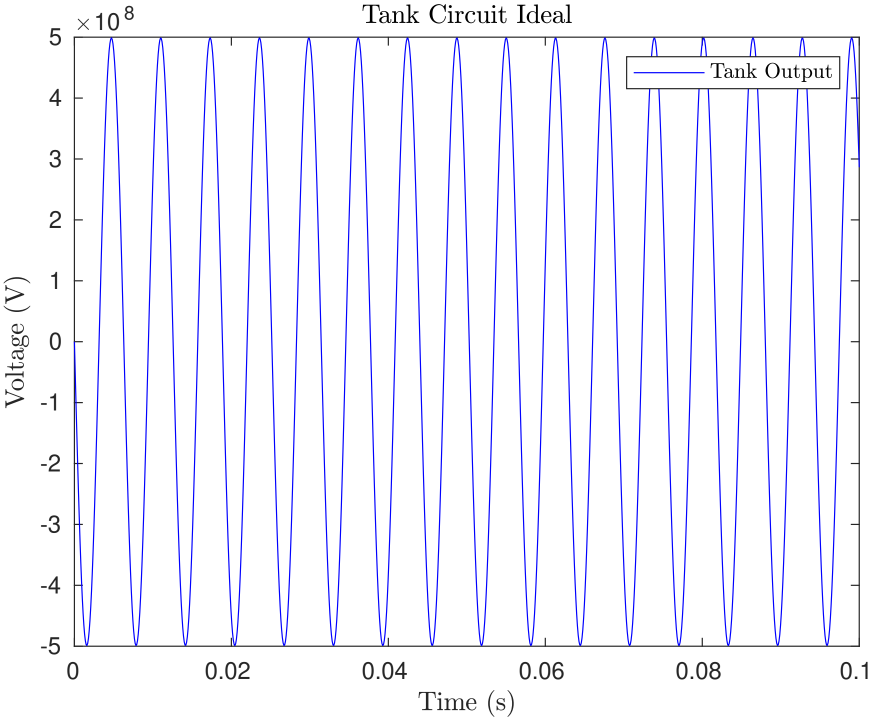

Figure 3: Plot of Tank Circuit

However, in the real world, components are non-ideal. Simulating the tank circuit with setting the series resistance of the inductor to a non-zero value, the wave decays as seen in Figure 4.

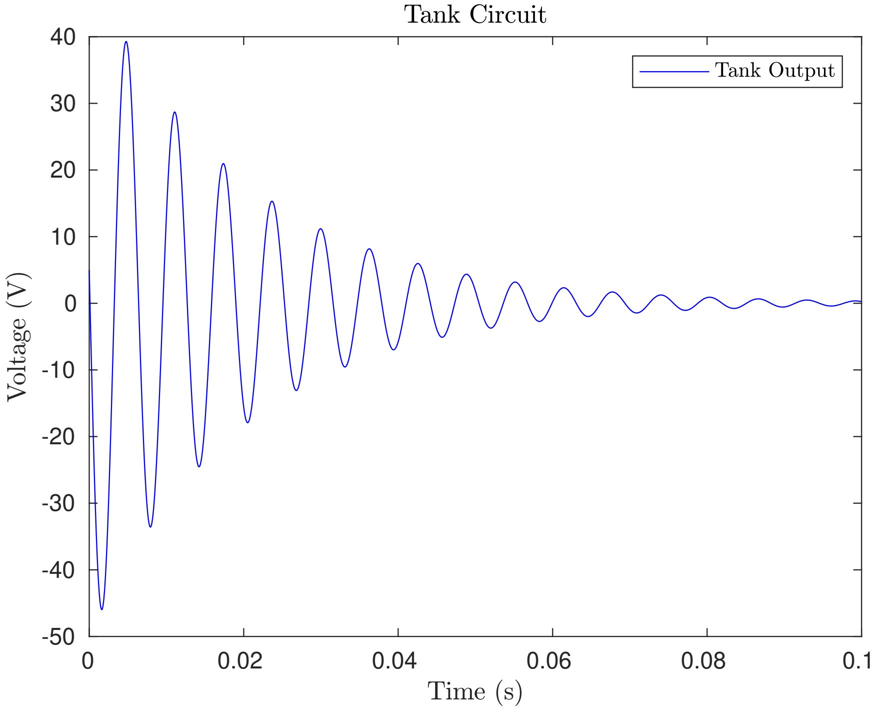

Figure 4: Plot of Tank Circuit

The resonant frequency f_r of this oscillator can be calculated via equation (1)

$$v f_r = \frac{1}{2\pi\sqrt{LC}} \tag{1}$$

It should be additionally noted that while the series resistance will not have an impact on the resonant frequency it will have an impact on the duration of the oscillations.

3.1.2 Operational Amplifier

Op-amp circuitry is added around the tank circuits to combat the decay. The op-amp uses a feedback loop to act as a current source to replenish the energy lost to the series resistance of the inductor[8].

3.1.3 LTSpice Implementation

Bringing together the tank the circuit with the op-amp feedback loop results in circuit seen in Figure 5.[10]

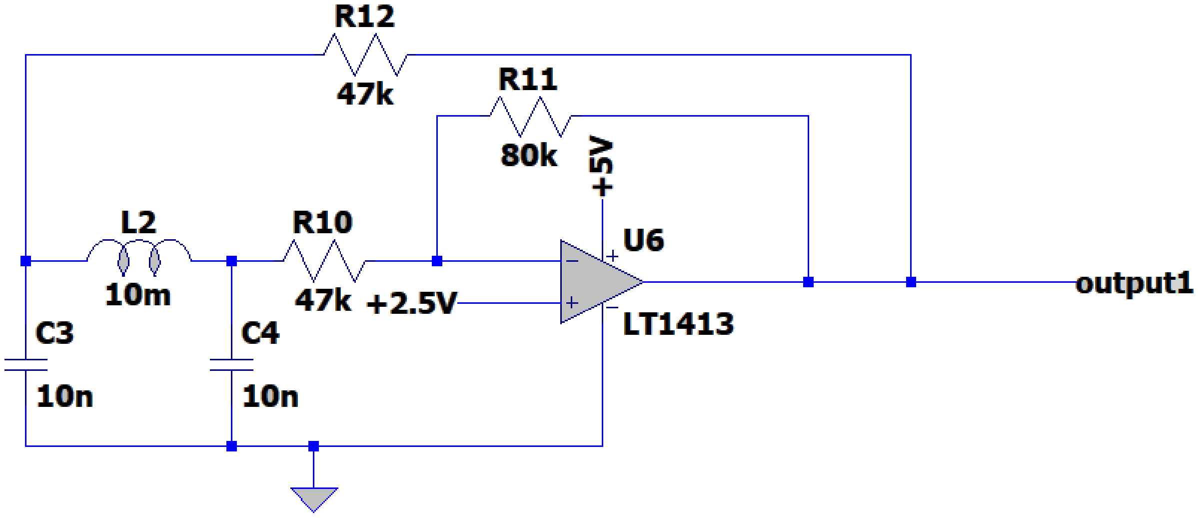

Figure 5: LTSpice Schematic of a Colpitts Oscillator

Simulating the complete circuit results in the plot seen in Figure 6. This will serve as one of the inputs to the mixers in the radio.

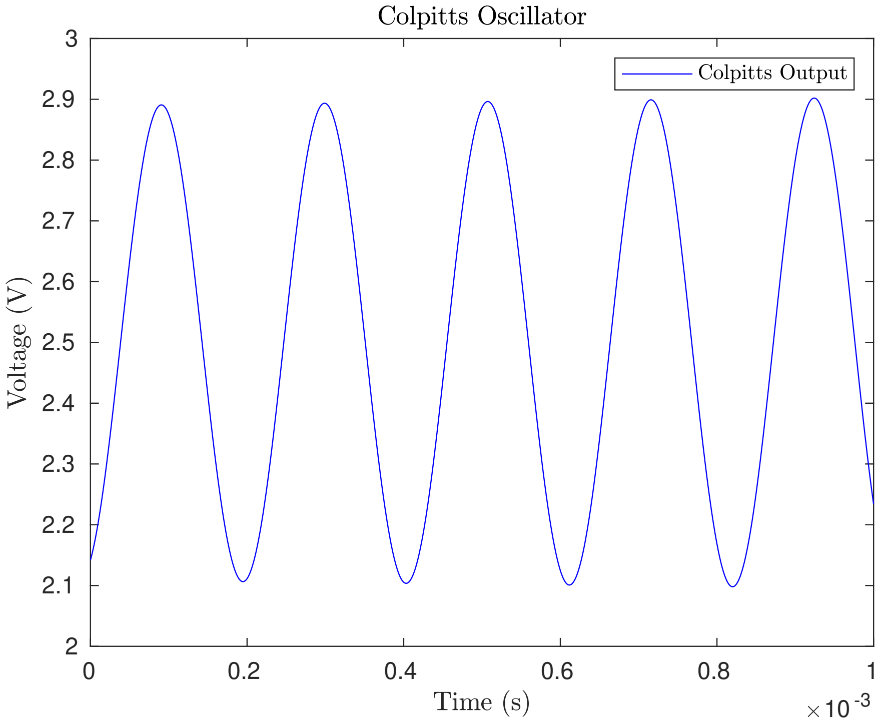

Figure 6: Plot of Colpitts Oscillator

3.2 Gilbert Cell Mixer

The Gilbert Cell mixer is a type of mixer that uses three differential amplifiers to produce an output signal of the product of two input signals [6].

3.2.1 Theoretical Behavior

A single differential amplifier uses two bipolar junction transistors (BJTs). BJTs function similarly to a switch; if the switch is open, there will be no current flowing through the transistor, however, if the switch is closed, the current will flow from the base to the emitter.

The circuit schematic shown in Figure 7 is an isolated differential amplifier. It takes in two input signals. These inputs are then subtracted using the transistor pair.

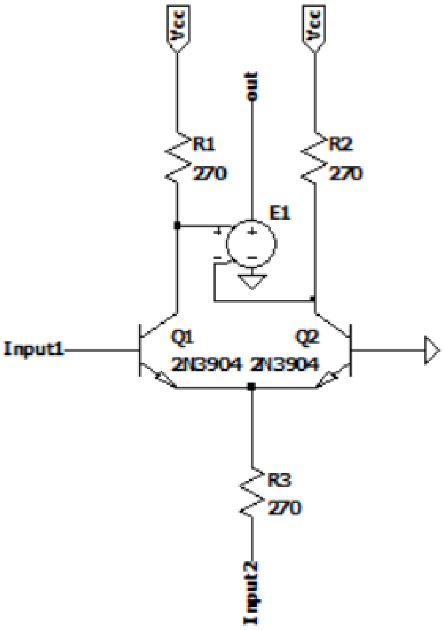

Figure 7: LTSpice schematic of only a single differential amplifier

The signal behavior of this circuit is shown in Figure 8. The plot shows the differential amplifier output voltage response when the inputs are two sine waves; one with a frequency of 1.1 kHz and the other with a frequency of 0.525 kHz.

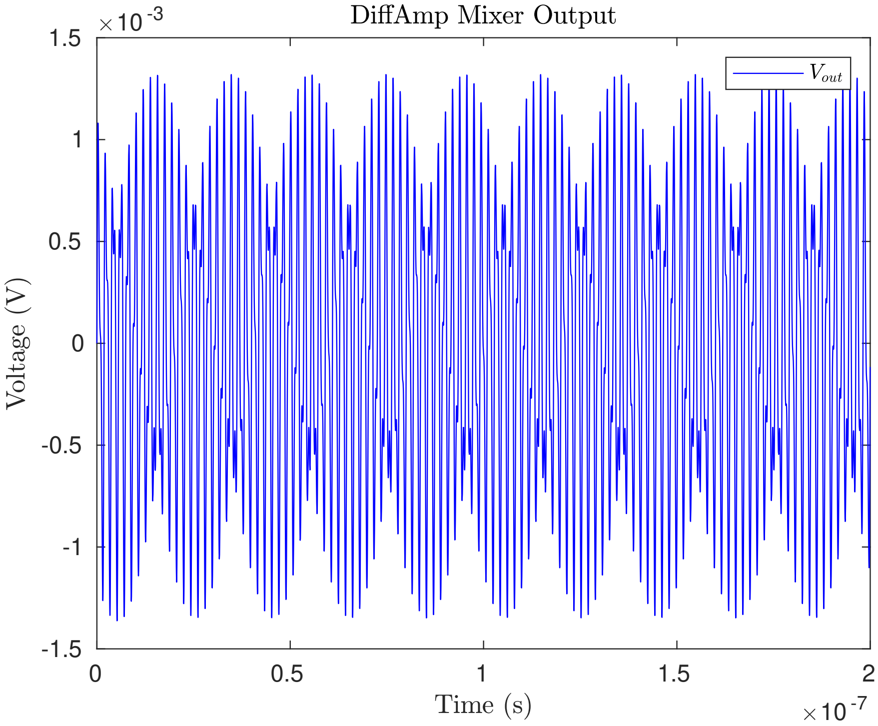

Figure 8: LTSpice output voltage waveform of a single differential amplifier

When looking at the signal behavior of this circuit, the two emitter currents I_{e1} and I_{e2} are given by

$$I_{e1} = \frac{I_0}{1 + e^{\frac{-V_i}{2}}} \tag{2}$$

$$I_{e2} = \frac{I_0}{1 + e^{\frac{V_i}{2}}} \tag{3}$$

where I_0 is the bias current which is defined as

$$I_0 = I_{e1} + I_{e2} \tag{4}$$

and V_i is the difference in between the input voltages V_{i1} and V_{i2} [7].

$$V_i = V_{i1} - V_{i2} \tag{5}$$

Using equations 2 and 3, when $\frac{I_{e1}}{I_0}$ is approximately one, then transistor one is operating in the active mode which means that it is operating similarly to a switch. This works similarly for transistor two's ratio $\frac{I_{e2}}{I_0}$.

When the first transistor *Q1* is turned ON, there will be a high voltage drop across *R3* and the collector of *Q1* will be less positive. Conversely, if *Q1* is OFF, the collector will be more positive.

This concept is applied to both transistors, and as a result, this amplifies the difference between the two input signals [2].

The output voltage for a differential amplifier, V_o, is the output signal response which is defined as

$$V_o = V_{c1} - V_{c2} \tag{6}$$

since the voltage of the combined signal response is the difference between the voltage across the first V_{c1} and second V_{c2} collector of the BJTs [5].

A single differential amplifier can only take the difference between two signals. To make a mixer, the signals need to be multiplied together. To solve this, three differential amplifiers can be combined to make a Gilbert Cell mixer.

Figure 9 shows the Gilbert Cell configuration consisting of differential amplifiers.

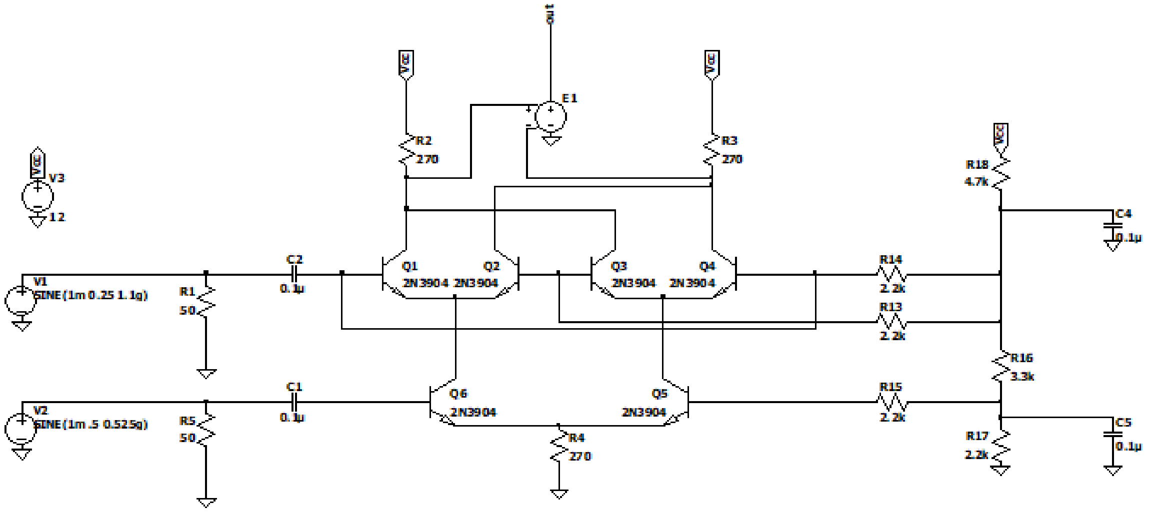

Figure 9: LTSpice Schematic of a Differential Amplifier Mixer

The outputs of the top two differential pairs, *Q1* and *Q2* as well as *Q3* and *Q4*, are connected together. The connected outputs of the differential pairs being connected will determine the voltage drop across *R2* and *R3*. This sums the two input signals.

The base of *Q1* is connected with *Q4*. The base of *Q2* is connected with *Q3*. When the currents of these top differential pairs are identical, the voltages dropped across *R2* and *R3* are identical. This is because as there is no difference between those two differential pairs, then the current through *Q6* is split evenly between the two collectors of *Q1* and *Q2*. This is the same for the current through *Q5*. If transistor *Q1* is completely ON and *Q2* is OFF, then *Q4* is ON and *Q3* is OFF. Since BJTs operate similarly to a switch, *Q2* and *Q4* are negligible since they don't have current flowing through them. If the current through *Q6* is the same as through *Q5*, then the difference between the two voltage drops is zero. However, if the currents are not equal, then the output voltage, *out*, will vary as a function of the input voltage *V1* which depends on the emitter currents of *Q5* and *Q6* [4].

The voltage input of the lower differential pair, transistors *Q5* and *Q6*, controls the current split between the emitter currents of the upper two differential pairs, *Q1* and *Q2* as well as *Q3* and *Q4*. Because of the lower differential pair, the output voltage also depends on the input voltage *V2*. This means that the output voltage V_{out} for the Gilbert Cell mixer is

$$V_{out} = a * V1 * V2 \tag{7}$$

where a is a scaling factor that depends on the gain created by the resistors *R2*, *R3*, and *R4*.

The circuit shown in Figure 10 ensures that the output signal's oscillations do not exponentially increase.

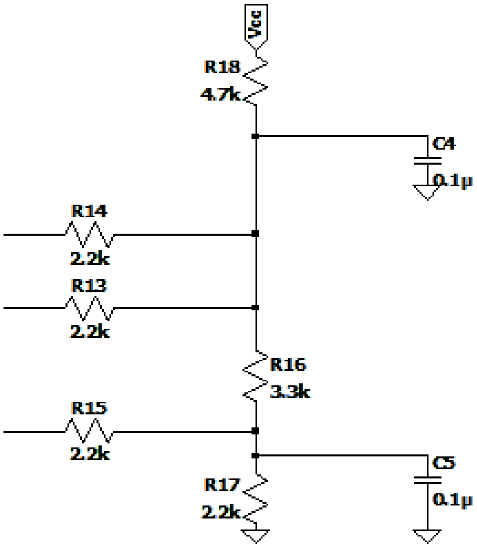

Figure 10: LTSpice schematic of the stabilizer

If the Gilbert Cell mixer did not have the stabilizer, the oscillations would compound and the signal's amplitude would amplify as shown in Figure 11.

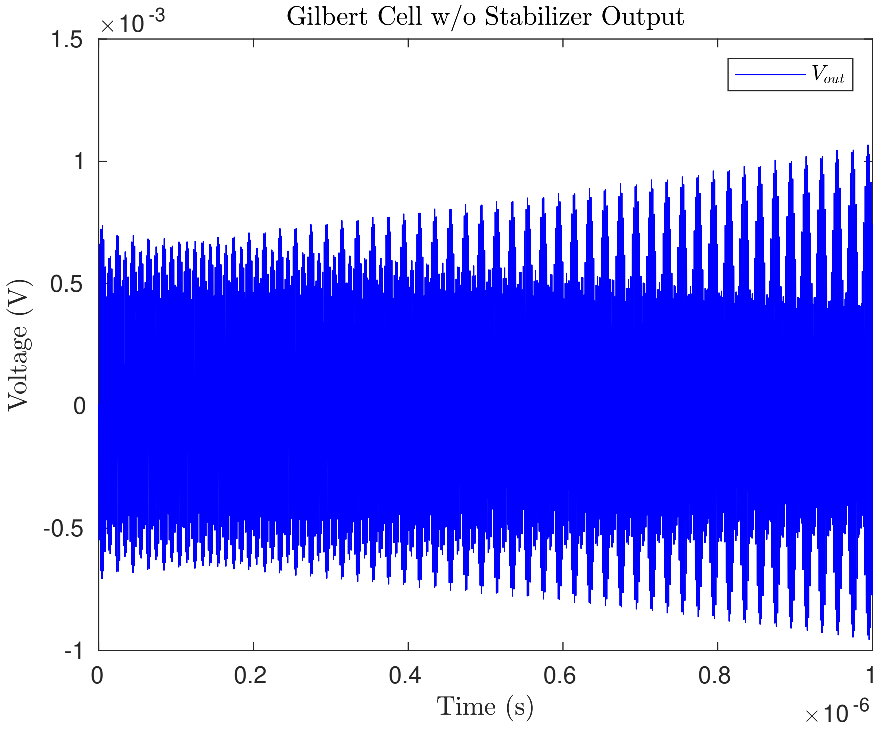

Figure 11: LTSpice waveform if the Gilbert Cell mixer without the stabilizer

This stabilizer circuit serves as a bias voltage for all the transistors in the differential pairs. Without this stabilizer, the closed loop nature of this circuit would cause a positive feedback loop, creating an output voltage signal shown in Figure 11.

The Gilbert Cell mixer utilizes differential amplifiers, as shown in Figure 9, to take in both inputs in the base of the *Q1* and *Q6*'s BJT. Then, the output signal is the voltage across collectors of the *Q1* and *Q4* BJTs.

The input signals for the Gilbert Cell mixer are sinusoidal. Since the voltage inputs are not constant, the output is dependent on the frequencies of the inputs. As such equation 7 can be re-written as

$$V_o = a * (\cos((w_c + w_m)t) + \cos((w_c - w_m)t)) \tag{8}$$

Where w_c is the carrier signal frequency (Hz) and w_m is the message signal frequency (Hz).

This is equivalent to taking the product of both input signals by using the cosine trigonometric identity

$$\frac{1}{2}\cos(w_c - w_m) + \frac{1}{2}\cos(w_c + w_m) = \cos w_c \cos w_m \tag{9}$$

The input signals for this circuit are two sinusoidal waves. The mixer circuit includes the compounded differential amplifiers and a stabilizer to output the product of the two input signals [12].

3.2.2 LTSpice Implementation

The implementation of the Gilbert Cell mixer in LTSpice worked as expected. When testing out the Gilbert Cell mixer, by itself, the output signal's amplitude would continue to grow by approximately 0.6 mV per 1000 nano-seconds. When the stabilizer was added, the output voltage amplitude did not increase. The implementation worked well, however, it worked better with higher frequency inputs, such as giga-Hertz compared to kilo-Hertz. When the input signals' frequencies were in the kHz range, the output voltage signal would initially increase and then oscillate around 80 mV.

Figure 12 shows the Gilbert Cell mixer signal output when the input voltage sine wave frequencies are 1.1 gHz and 0.525 gHz.

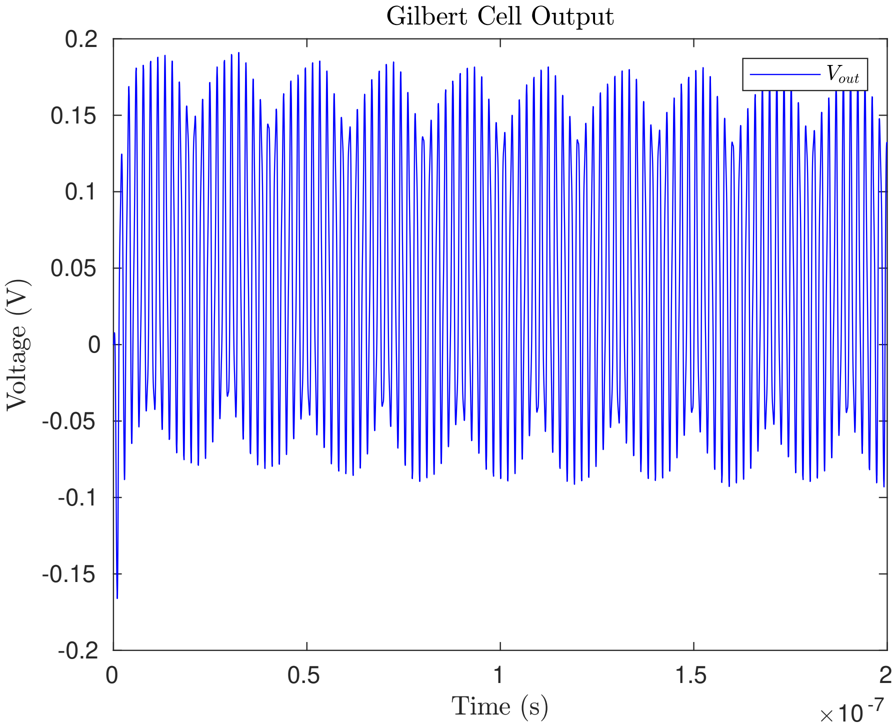

Figure 12: LTSpice waveform of the Gilbert Cell mixer implementation

3.3 Log Anti-Log Mixer

A logarithmic, or log, amplifier is a circuit that takes the logarithm of the applied input using a resistor, op-amp, and diode (Figure 13). An op-amp is an active component that takes two inputs, compares them, then produces an output. Oftentimes, op-amps are used in positive or negative feedback to add, subtract, amplify, etc [8].

3.3.1 Theoretical Behavior

For the log amplifier, the non-inverting input of the op-amp is wired to ground, and by using the concept of a virtual short, the voltage at the inverting input can be assumed to match the voltage of the non-inverting input, zero Volts [11]. Knowing this and that inputs of an op-amp do not sink any current, the voltage of the output can be calculated as:

$$V_{out} = -nV_T \ln(\frac{V_{in}}{RI_s}) \tag{10}$$

where n is the ideality factor of the diode or how close to ideal it behaves, V_T is the diode's thermal voltage, and I_s is the diode's saturation current. Although the output is scaled by R and the diode constants, and then inverted, it can be seen that the diode takes the natural logarithm of the input V_{in}.

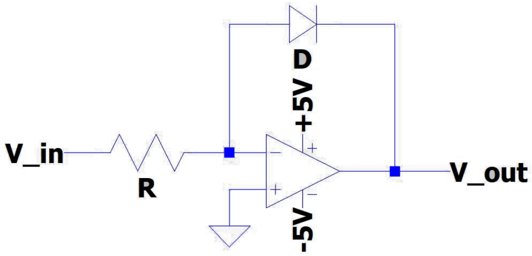

Figure 13: LTSpice schematic of an logarithmic amplifier using an op-amp

An anti-logarithmic, or anti-log, amplifier is a circuit that takes the anti-logarithm of the applied input using a resistor, op-amp, and diode (Figure 14). Similar to a log amplifier, the the non-inverting input of the op-amp is wired to ground, allowing the concept of a virtual short to be applied [11]. Therefore, the voltage of the inverting input can be assume to be equal to that of the non-inverting input or zero Volts. Knowing this and that inputs of an op-amp do not sink any current, the voltage of the output can be calculated as:

$$V_{out} = -RI_s e^{\frac{V_{in}}{nV_t}} \tag{11}$$

Although the output is scale and inverted, the anti-log amplifier takes the anti-log or e to the power of the input signal scaled by some factor.

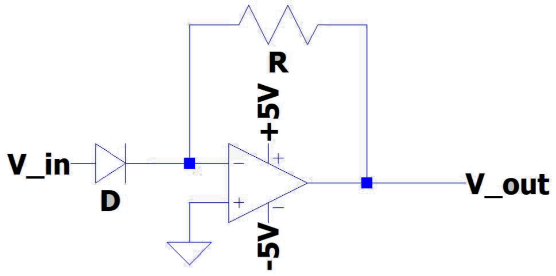

Figure 14: LTSpice schematic of an anti-logarithmic amplifier using an op-amp

An adder is another op-amp circuit used to sum two input signals using three resistors (Figure 15). Similar to the log and anti-log amplifiers, the concept of a virtual short must be applied to analyze the circuit [8]. By utilizing that concept and Ohm's Law, the following equation can be found to determine the voltage out V_{out}:

$$V_{out} = -\frac{R_F}{R_{IN}}(V_1 + V_2) \tag{12}$$

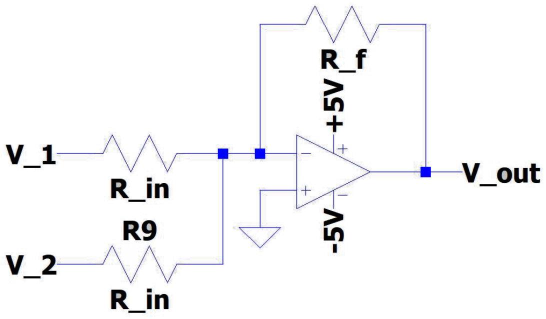

Figure 15: LTSpice schematic of an op-amp inverting adder

In addition to inverting adders, non-inverting adders are also a useful tool to add DC offsets and amplify signals. Figure 16 contains a circuit diagram of a non-inverting op-amp, which no longer uses a virtual short and instead uses Kirchoff's laws to produce the equation:

$$V_{out} = (1 + \frac{R_A}{R_B})\frac{V_1 + V_2}{2} \tag{13}$$

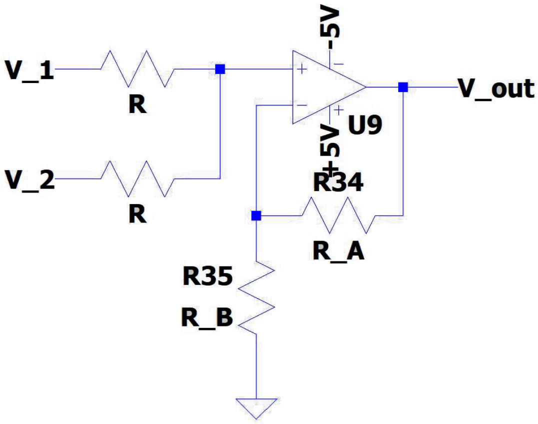

Figure 16: LTSpice schematic of a non-inverting op-amp adder

This adder adds two signals in addition to scaling the output, which can be used to add a DC offset or scale an output.

By combining an inverting adder, two log amplifiers and one anti-log amplifier in the correct configuration, a multiplier can be created. First, the logarithm of the message signal, V_m, and carrier signal, V_c, is taken, then added. This results in:

$$-nV_T\ln(\frac{V_c}{RI_s}) + -nV_T\ln(\frac{V_m}{RI_s}) \tag{14}$$

Which can be simplified using properties of logarithms assuming identical resistors and diodes are used:

$$-nV_T\ln(\frac{V_c}{RI_s}\frac{V_m}{RI_s}) \tag{15}$$

Finally, the added signal is then passed through an anti-log amplifier, resulting in the following equations. The following equations walk through simple algebraic simplifications.

$$V_{out} = -RI_s e^{-\frac{1}{nV_t}\cdot -nV_T\ln(\frac{V_c}{RI_s}\frac{V_m}{RI_s})} \tag{16}$$

$$V_{out} = -RI_s e^{\ln(\frac{V_c}{RI_s}\frac{V_m}{RI_s})} \tag{17}$$

$$V_{out} = -RI_s(\frac{V_c}{RI_s}\frac{V_m}{RI_s}) \tag{18}$$

$$V_{out} = -\frac{V_c V_m}{RI_s} \tag{19}$$

This ultimately results in a mixed signal that is scaled and inverted. To ensure V_{out} is centered at the appropriate voltage and large enough to be detected, an additional inverting adder can be used as seen in Figure 17. This adder first centers the signal close to zero Volts then inverts it to result in a more usable signal.

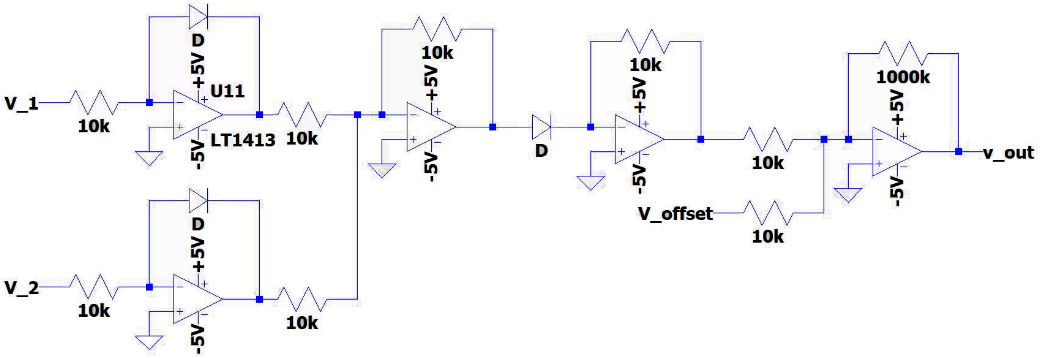

Figure 17: LTSpice schematic of a log anti-log mixer

3.3.2 LTSpice Implementation

The implementation of a log anti-log mixer in LTSpice worked as expected, although extra care was necessary to ensure that the rails of the op-amp were not hit. This is because each adder, log amplifier, and anti-log amplifier inverts the signal. Before this was accounted for, the circuit would constantly rail to zero Volts due to the nature of op-amps. Additionally, the saturation current, I_s of the simulated diode were relatively large on the order of magnitude 10^{-4} amps. This made it very important to scale input signals appropriately to ensure outputs fell within a reasonable range. Furthermore, because of the nature of logarithms, the input signals must be positive to prevent sawtooth or other unwanted circuit behavior. Figure 18 shows the circuit with all resistor values.

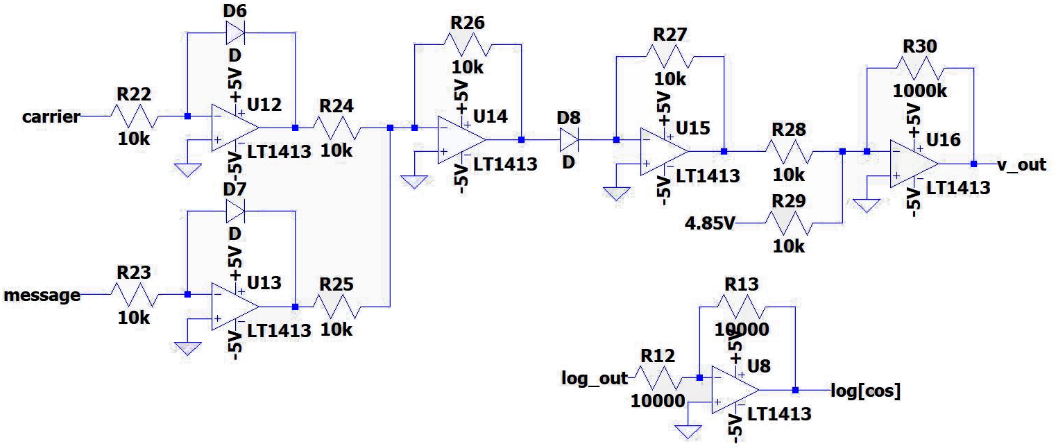

Figure 18: LTSpice schematic of a log anti-log mixer implementation

By passing two sinusoids–one carrier with a frequency of approximately 15,000 Hz and an amplitude of 0.8 V and a message with a frequency of 2,000 Hz and amplitude of 0.3 V– into the circuit seen in Figure 18, an amplitude modulated signal is produced (Figure 19). The modulated signal, however, has a very small amplitude due to the RI_s term seen in equation 19.

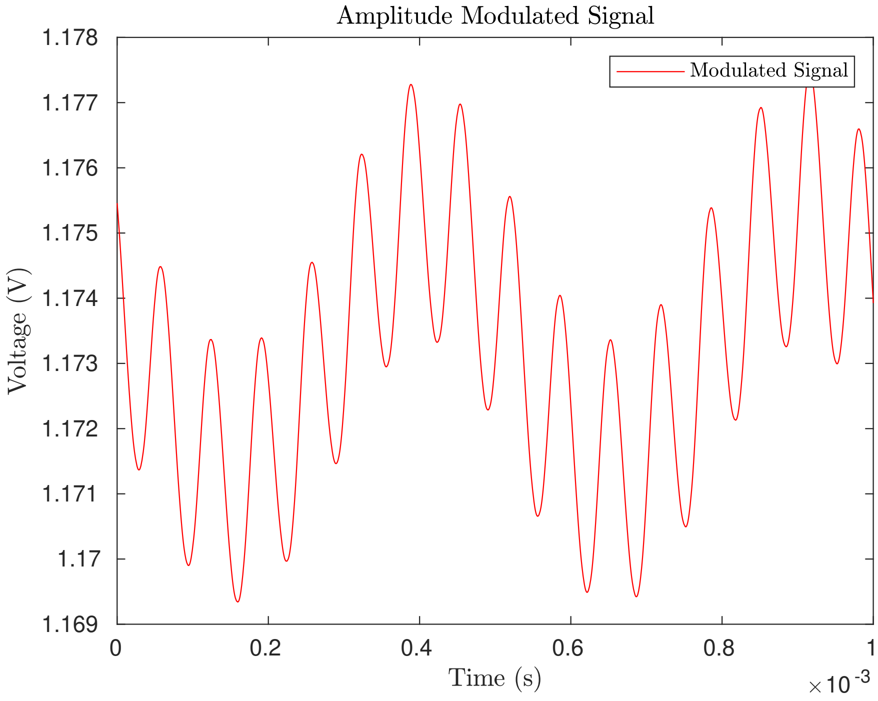

Figure 19: Plot of amplitude modulated signal

4 Receiver

The receiver, like the transmitter, is built using a few components. In many cases, an antenna or something similar is needed to receive the signal, but for the tests in this paper, the transmitter was connected via a wire. Because an amplitude modulated signal is being received, the envelope of that signal must be extracted to find the original message signal. This is often accomplished by using a rectifier paired with a low pass filter.

4.1 Envelope Detector

An envelope detector, sometimes called a peak detector, extracts the peak amplitudes of a high frequency signal [1].

4.1.1 Theoretical Behavior

For the scope of this paper, the rectifier was composed of a singular diode, which zeros out all negative components of the modulated signal, assuming the modulated signal is centered at zero Volts. This results in a signal containing the envelope encoded onto the positive part of the carrier wave. Filtering, namely low pass filtering, following the rectifier completes envelope detection by smoothing out the high frequency carrier wave and leaving only the messenger wave untouched [1]. This and other methods of envelope detection, however, are not perfect and produce a jagged signal that must be smoothed using an additional low pass filter. The circuit seen in Figure 20 is a simple envelope detector.

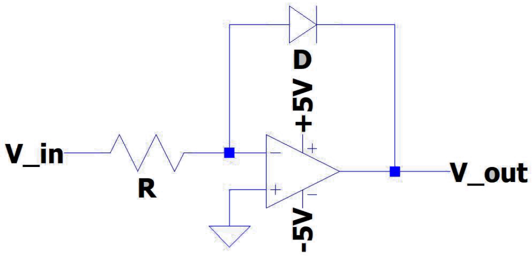

Figure 20: LTSpice schematic of an op-amp adder

4.1.2 LTSpice Implementation

The envelope detector worked as expected, but because the input modulated signal had such a small amplitude, additional gain of 300 was needed on the output. The demodulated signal before and after being amplified can be seen in Figure 26, where the signal amplification has an amplitude of one milli-Volts. The signal after amplification is much more usable with an amplitude of 200 mV. Furthermore, there is clear distortion present in the demodulated signal, which could, in the future, be mitigated using additional filtering.

5 Results

In integrating the Colpitts oscillator with a log anti-log mixer and envelope detector, a full, simulated radio can be created, where the channel is a wire. The entire circuit can be seen in Figure 25. The Colpitts oscillator first creates a carrier wave with a frequency of $\frac{1}{2\pi\sqrt{LC}}$, where C is the equivalent capacitance of the tank circuit. This produces a carrier frequency of 15,175 Hz. A DC offset is then added to the signal to center the carrier at 2.5V as seen in Figure 21. If the DC offset is not added, the following log amplifier would produce a less than ideal output due to the negative part of the signal.

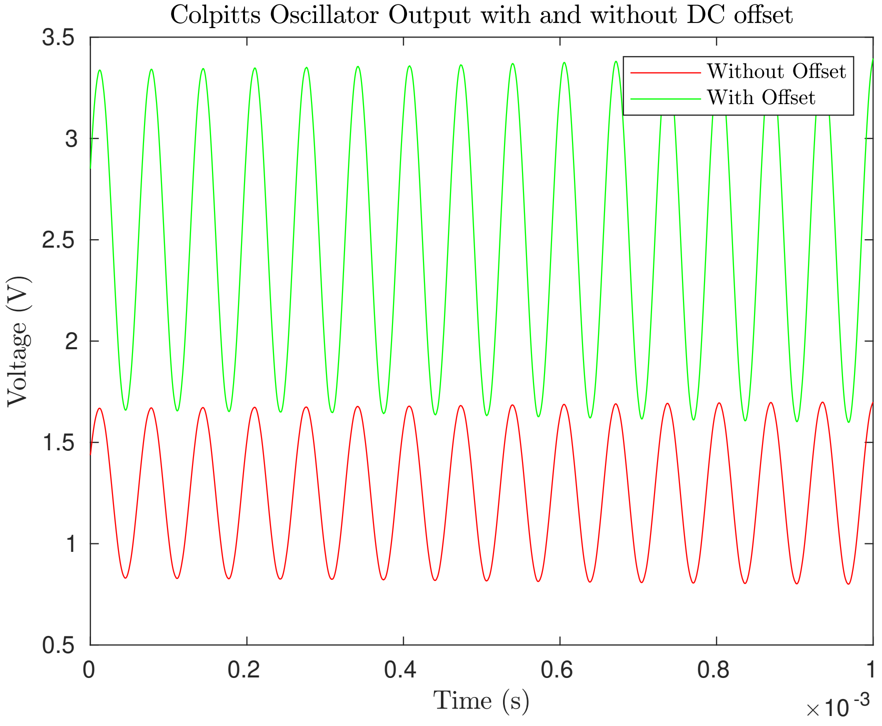

Figure 21: Plot showing an output of a Colpitts oscillator with and without DC offset

For the first attempt, the carrier was modulated with a message signal using the Gilbert Cell mixer. The schematic of the combined Colpitts oscillator, Gilbert Cell mixer, and envelope detector are shown in Figure 22.

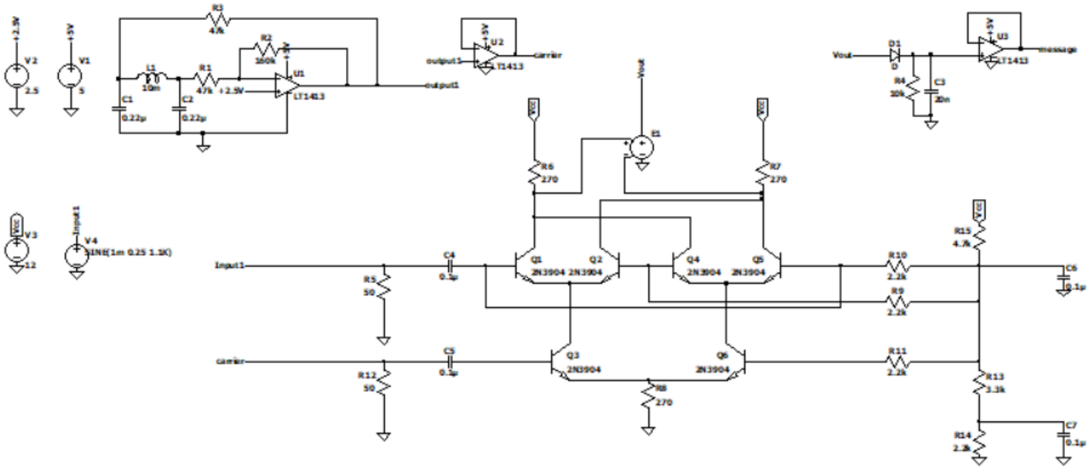

Figure 22: LTSpice schematic of combining the Colpitts oscillator, Gilbert Cell mixer, and envelope detector

Although the Gilbert Cell mixer worked independently, when integrating it with the other components, there was no modulation occurring between the input signals.

Since the the Gilbert Cell attempt did not work, a log anti-log mixer was used. In this case, the message signal is a sinusoid with a frequency of 2000 Hz and an amplitude of 0.3 V centered at 2.5 V. These input signals can be seen in Figure 23. The mixer ultimately produces an amplitude modulated signal centered at 1.73 V with an amplitude of approximately 60 mV (Figure 24). This modulated signal was then sent through a wire channel with zero resistance.

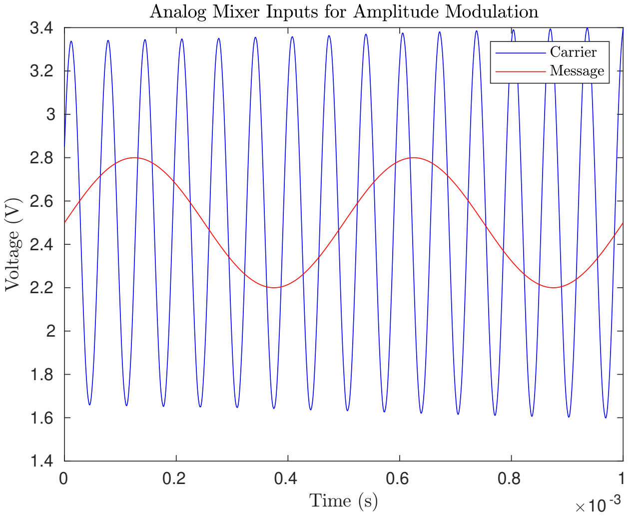

Figure 23: Possible inputs into a mixer for amplitude modulation

Figure 24 shows the output of a carrier signal and message in Figure 23 mixed together to produce a modulated signal.

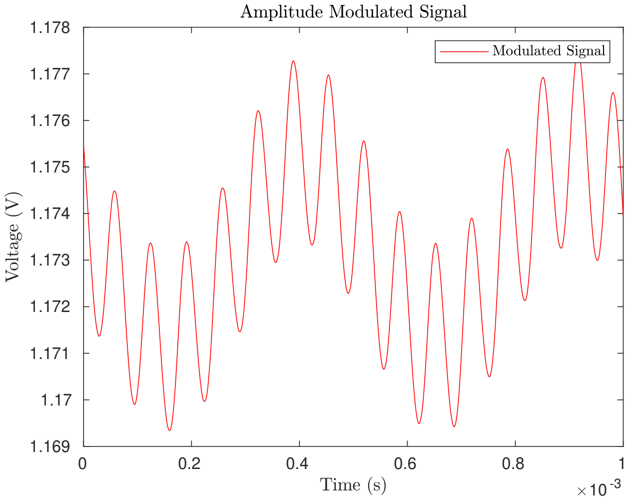

Figure 24: Amplitude modulated signal produced by mixing a carrier and message

Figure 24 demonstrates the low frequency signal wave being encoded as the peak amplitude of the high frequency carrier wave.

The entire circuit configuration can be seen in Figure 25. It shows the combined Colpitts Oscillator, Log-Antilog Mixer, and the Envelope Detector circuits.

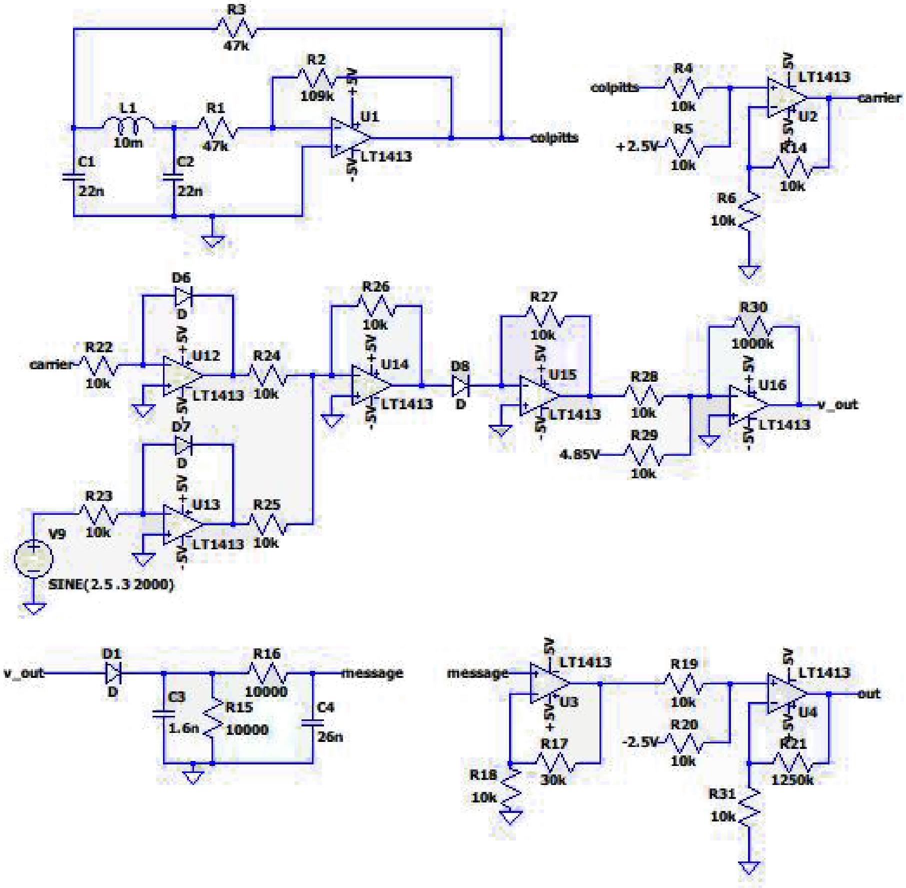

Figure 25: LTSpice schematic of combining the Colpitts oscillator, log antilog-mixer, and envelope detector

The received signal was demodulated using the envelope detector described in section 4.1. This produces the original message signal with an amplitude of 150 mV as seen in Figure 26. There is however some distortion present in the signal due to the nature of filtering.

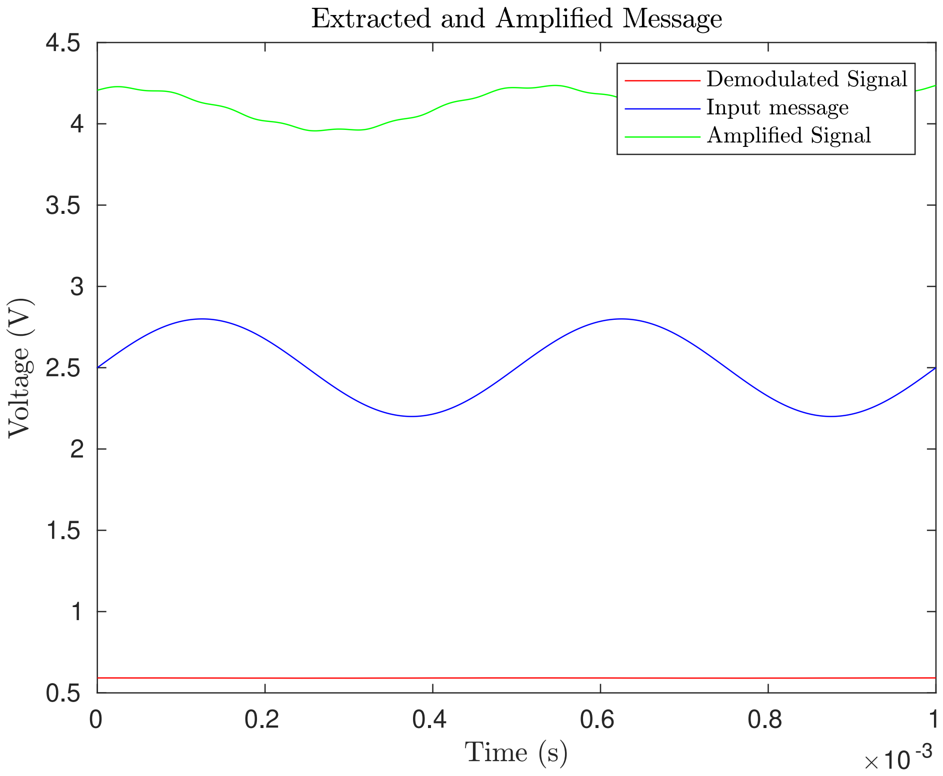

Figure 26: LTSpice schematic of combining the Colpitts oscillator, log antilog-mixer, and envelope detector

Though this circuit works, it has many components specifically selected for the given carrier frequency and voltage inputs. As a result, any change in carrier or inputs would require a large amount of circuit analysis to adjust DC voltage offsets and gains to produce a usable signal. This is largely the case due to the limitations of an op amp in conjunction with the nature of logarithmic amplifiers.

6 Future Work

Further iterations of this project include implementing the simulated circuits onto a breadboard. By building the circuits on a breadboard, real-world constraints can be tested and validated. Next, the team would also develop other models and compare the signal quality between these different models to determine which combinations work best. Once the final designs for each of the sections in the system is finalized, the next step would be to make a printed circuit board (PCB).

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