Passive Solar Tiny Home for Massachusetts

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Abstract

For the following project, we were asked to create a tiny solar house that would allow individuals in Boston to live at a comfortable temperature(between 18-25 degrees Celsius or 291.15-298.15 Kelvin) during the winter season. Restrictions included that the dimensions of the house must be between 100 and 400 square feet and it cannot receive direct sunlight through the windows till noon in the summer. Our goal was to design a lofted house that was both aesthetically pleasing and sustainable. We considered materials like hempcrete and 3D printer extrudable concrete; and since they were reusable, recyclable and less toxin in comparison to other materials. We expect heat to rise in the two-story house, however, for our model, we are deciding to implement a uniform air temperature.

Background

In the Greater Boston Area, the annual energy consumption and carbon footprint, is "6.1 million metric tons of greenhouse gases... with residential buildings accounted for 19% of emissions" according to "Boston's Carbon Emissions." While the Boston's Carbon Emission article does mention that there was a "21% decrease from 2005" in emissions, the current warming trend is still an issue. *According to NASA's Climate Change: How Do We Know?* article, the Earth's climate changing rapidly with "the planet's average surface temperature has risen about 1.62 degrees Fahrenheit ... since the late 19th century, a change driven largely by increased carbon dioxide and other human-made emissions into the atmosphere."[1] Since global warming will continue to advance because of man-made emissions, the need for sustainable lower-energy housing is needed to decrease emissions further.

[1]Changed quote to develop our argument as suggested

1 Process

1.1 Design

Heading into this project, we had two objectives for the design of our solar home: create a house that was two stories tall and was composed of unique sustainable materials. The first material we chose was extrudable concrete, which is a lightweight concrete used in construction based 3D printing for buildings. The second material is hempcrete, which is said to be effective at thermal regulation and is made comprised of hemp shiv, binder, natural hydraulic lime, and a little bit of concrete. In order to make our house two stories, we extended the overhang on the roof for the summer sun position, and essentially stacked two smaller homes on top of each other, which would increase sun exposure in the winter by making the house taller, rather than longer.

The reason we chose extrudable concrete is because of its versatility and environmental sustainability. It allows for homes to be made rapidly, at low costs, and with creative designs. In comparison to concrete, both are relatively sustainable, but extrudable concrete is a lightweight concrete and increases the efficacy and creativity of home/building design.

According to American Lime Technology, Hempcrete is based from the woody core of the hemp plant, and has little to no THC in it despite it being a cousin of the marijuana plant. Hemp's potential lies in the fact that it is a weed killer, pesticide, fungicide, and requires no fertilizer. This both protects other crops and makes its carbon footprint smaller due to its lower demand for chemicals and manufacturing. It's been used since the 6th century and is resurfacing now due to its thermal and sustainable properties. Since it's lighter than concrete, it can be moved very easily by workers on site. The caveat is that hemp is illegal to grow in the United States despite its high construction potential, this increases its carbon footprint due to UK imports.

We based our house on the sample design given in the project descriptions and we modified it to meet our goals. In order to make our house two stories tall, we extended the overhang on the roof for the summer sun position, and essentially stacked two smaller homes' heights on top of each other, which would increase sun exposure in the winter by making the house taller, rather than longer.

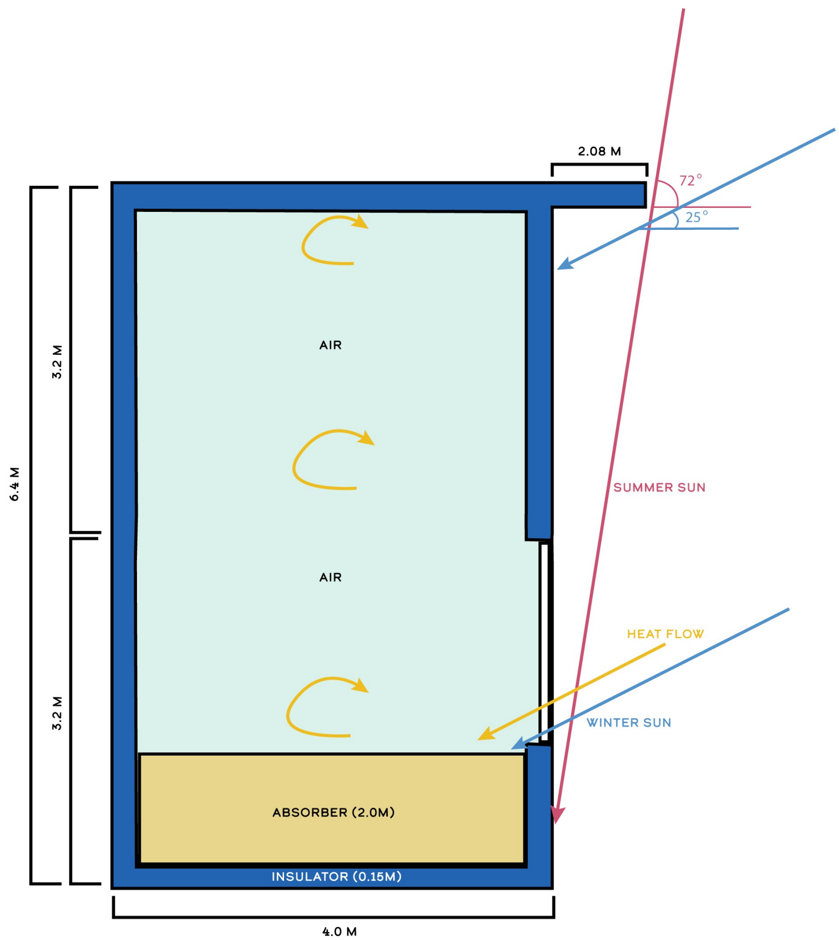

Figure 1: This figure shows the diagram of our passive solar house.

Displayed in Figure 1[2] is an image of our design. As can be seen, there's insulation (extrudable concrete) that serves as the walls. There is also an absorber layer (hempcrete). The dimensions of our solar home are 4 x 4 x 6.4 (in meters). The floor space of our home is approximately 172 meters squared.

1.2 Model

1.2.1 Assumptions

We made several assumptions for our model:

[2]Updated figure to represent our new house design as suggested.

- Heat loss is only through the walls, and not through the window.
- All energy coming through the window is absorbed by the absorber.
- The inside air has a uniform temperature.
- The walls and the absorber also have a uniform temperature

The table below shows the variable definitions for our equations. Values in the Value column are NaN when we are solving for those values for our model of our house.

Variable Definitions			
Variable Name	Definition	Values if applicable	Units
T_{out}	Temperature of Outside Air	270	K
T_{wall}	Temperature of Wall	NaN	K
T_A	Temperature of Absorber	NaN	K
T_a	Temperature of Internal Air	NaN	K
$L_{absorber}$	Length of Absorber	2.2	m
$L_{insulation}$	Length of Insulation	0.15	m
$A_{absorber}$	Area of Absorber	16	m^2
A_{wall}	Area of Wall	53	m^2
$D_{absorber}$	Density of Absorber	275	Kg/m^3
D_{wall}	Density of Wall	2168	Kg/m^3
$c_{absorber}$	Specific Heat of Absorber	1600	J/KgK
c_{wall}	Specific Heat of Wall	800	J/KgK
h_{air}	HeatTransfer Coeff of both Inside & Outside Air	20	W/m^2K
$k_insulation$	Thermal Conductivity of Insulation	0.3	W/mK

Equation definitions are used for our main ordinary differential equations of our model.

Equation Definitions		
Equation	Definition	Units
$Q_{sun} = (-0.3416 * cos(t * \pi/43200)+$ $0.2243 * cos(t * \pi/(21600)) + 0.2097)$	Energy Input from Sun	W
$Q_{in} = Q_{sun} * A_{Window} * 1000$	Energy Input from Sun Through Window	W
$m_{absorber} = L_{absorber} * D_{absorber} * A_{absorber}$	Mass of Absorber	Kg
$m_{wall} = L_{insulation} * D_{insulation} * A_{wall}$	Mass of Wall	Kg
$C_1 = m_{absorber} * c_{absorber}$	Capacitance of Absorber	J/K
$C_2 = m_{wall} * c_{wall}$	Capacitance of Wall	J/K
$R_1 = 1/(h_{air_{in}} * A_{absorber})$	Resistance between Inside Air and Absorber	W/K
$R_2 = 1/(h_{air_{in}} * A_{wall})$	Resistance between Inside Air and Wall	W/K
$R_3 = L_{insulation}/(k_{insulation} * A_{wall})$	Resistance between Inside and Outside of Wall	W/K
$R_4 = 1/(h_{air_{out}} * A_{wall})$	Resistance between Outside Air and Wall	W/K

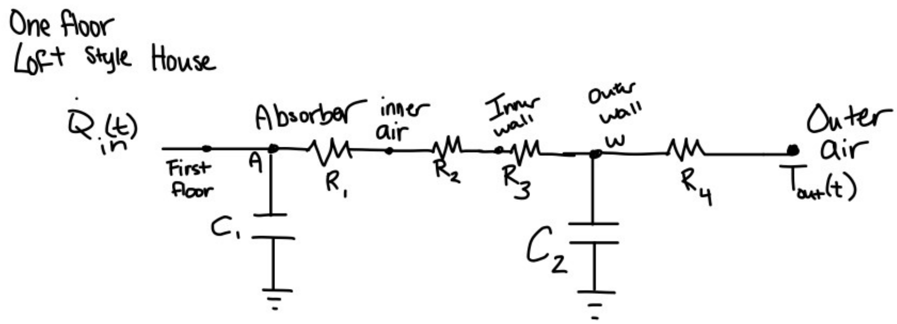

Figure 2: This figure shows the circuit representation of our model for our lofted style house.

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We modeled the heat transfer of the first floor of our house through a circuit diagram shown in Figure 2.

$$C_1 \frac{\mathrm{d}T_A}{\mathrm{d}t} = Q_{in}(t) - \frac{T_A - T_{wall}}{R_1 + R_2 + R_3} \tag{1}$$

$$C_2 \frac{\mathrm{d}T_w}{\mathrm{d}t} = \frac{T_A - T_{wall}}{R_1 + R_2 + R_3} - \frac{T_{wall} - T_{out}}{R_4} \tag{2}$$

$$T_a = T_{wall} + \frac{(T_A - T_{wall})(R_2 + R_3)}{R_1 + R_2 + R_3} \tag{3}$$

We symbolically represented the energy balance on the absorber with Equation (1) and the energy balance on the wall with Equation (2). Equation (1) took into account the energy input from the sun and the temperature of the absorber as a function of time represented as a capacitor. Equation (2) modeled the temperature of the wall as a function of time represented as a capacitor. The internal air and inner wall were represented as resistors since they are just a medium in which heat flows through. Whereas the absorber and outer wall are represented as capacitors and resistors since they both store heat and are also a medium in which heat flows through. We then used the values of T_A and T_{wall} we got from Equations (1) and (2) to find the temperature of the air with Equation (3). We used this internal temperature to gauge whether our customized values for $L_{insulation}$ and $L_{absorber}$ would get us in our desired internal air temperature range (18-25 ° C). [3]

[3] Explained the equations more as suggested.

1.3 Model Results

Our plot in Figure 3 shows the internal air temperature over 24 hours for days 8 to 9. We chose several days after the beginning our model because we experimentally saw that over time, the temperature began to stabilize and we would be able to get more accurate predictions of the house's internal temperature as a whole instead of only when the house is finished being built.

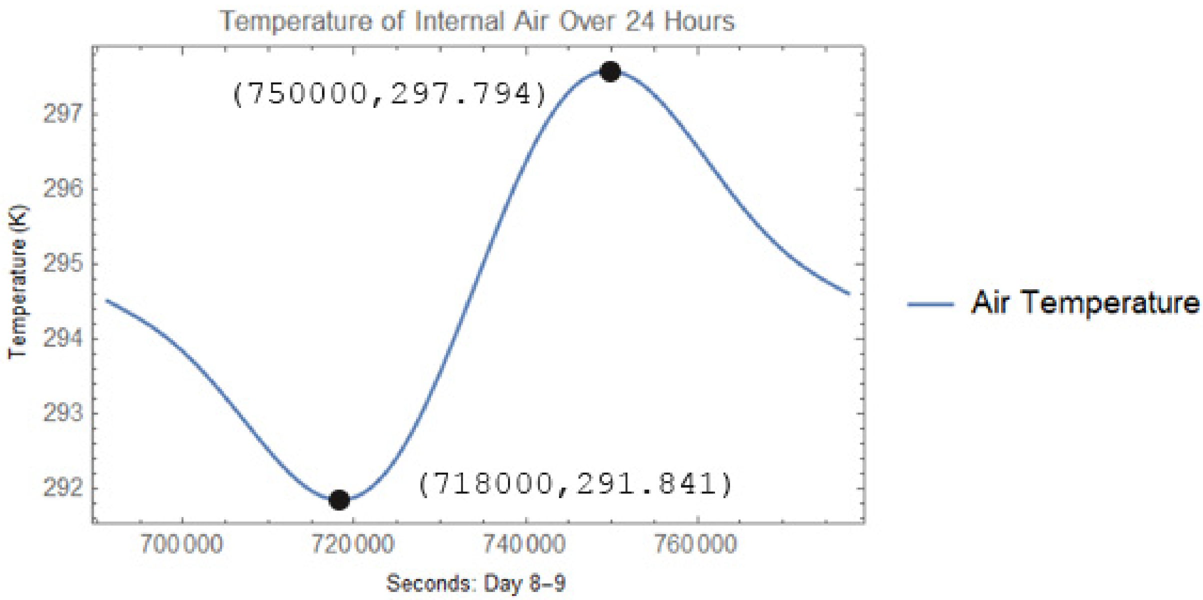

Figure 3: This figure shows the periodic plot of the temperature of internal air over 24 hours with labeled minimum and maximum points.

The plot in Figure 4 shows the temperature of our absorber, walls, and internal air with our chosen values of $L_{insulation}$ and $L_{absorber}$ to reach our desired range for internal air. The maximum and minimum air temperatures were recorded as approximately 297.574 degrees Kelvin (24.841 degrees Celsius) and 291.841 degrees Kelvin (18.8407 degrees Celsius) respectively, which fall within the 18-25 degrees Celsius range of "comfortable temperature".

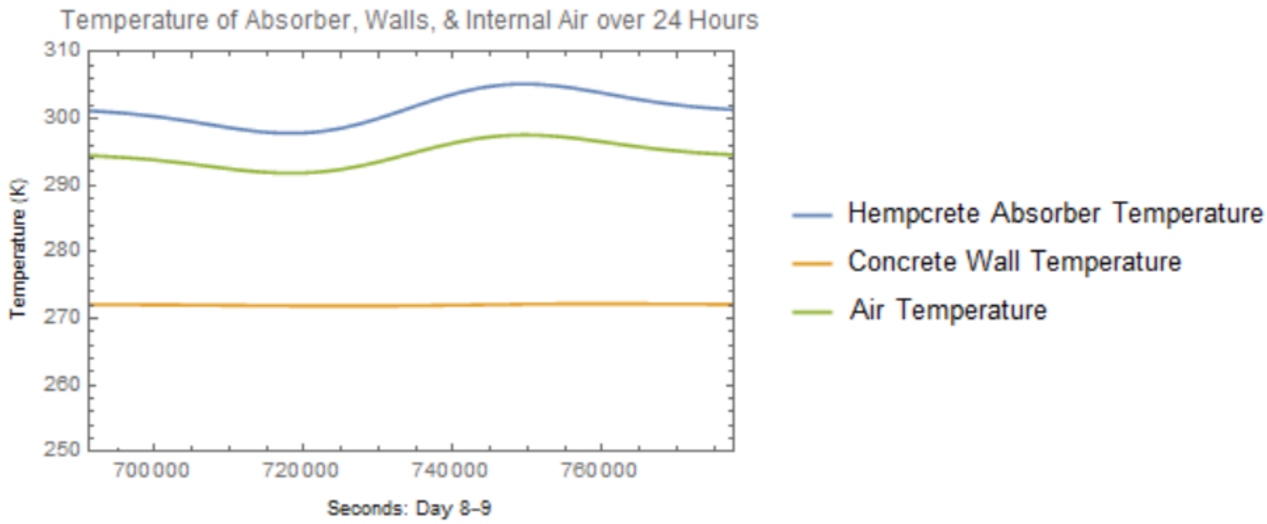

Figure 4: This figure shows the periodic plot of the temperature of the absorber, walls, and internal air over 24 hours with our experimentally validated values for $L_{insulation}$ and $L_{absorber}$.

Increasing the insulation thickness trapped more heat in the house, which meant that the overall temperature of the house would increase over time as shown in Figure 5. Decreasing the insulation thickness of the walls will not trap the heat since, going back to our circuit model, the resistance value of the walls will be small, so the voltage drop will not be as big of a difference and heat would just flow through as shown in Figure 6.

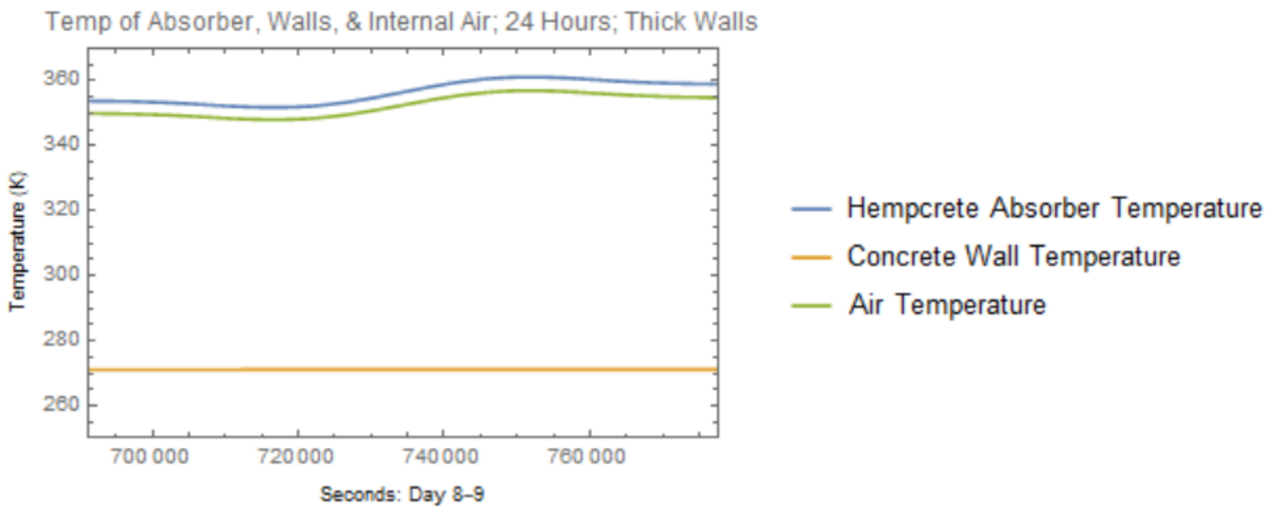

Figure 5: This figure shows the periodic plot of the temperature of the absorber, walls, and internal air over 24 hours with 1 meter for wall thickness.

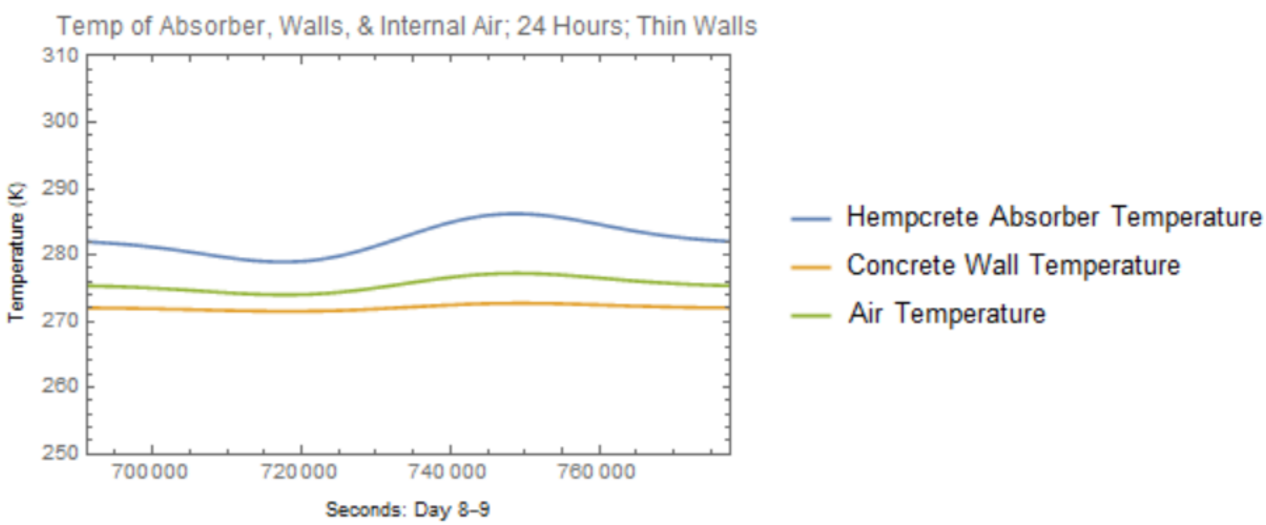

Figure 6: This figure shows the periodic plot of the temperature of the absorber, walls, and internal air over 24 hours with 0.01 meters for wall thickness.

Decreasing the absorber thickness will make the air have almost the same temperature as the absorber. Since the absorber is like a capacitor and it stores heat, the thinner the absorber, the less heat it can store. This means that the heat going in to the thinner absorber would be almost instantly released to the internal air, thus making the internal air temperature approximately the same as the absorber as shown in Figure 7. Increasing the absorber thickness will make the air temperature rise slowly over time. This is better in the long run, especially in the winter season, since the absorber will store energy and slowly warm the inside of the house as shown in Figure 8.

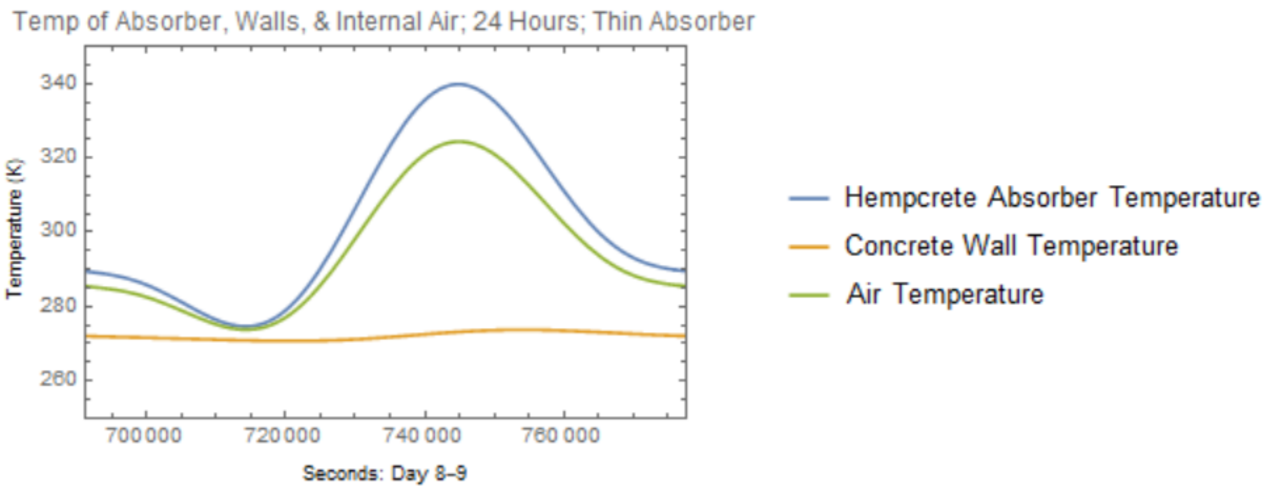

Figure 7: This figure shows the periodic plot of the temperature of the absorber, walls, and internal air over 24 hours with a thinner absorber of 0.2 meters.

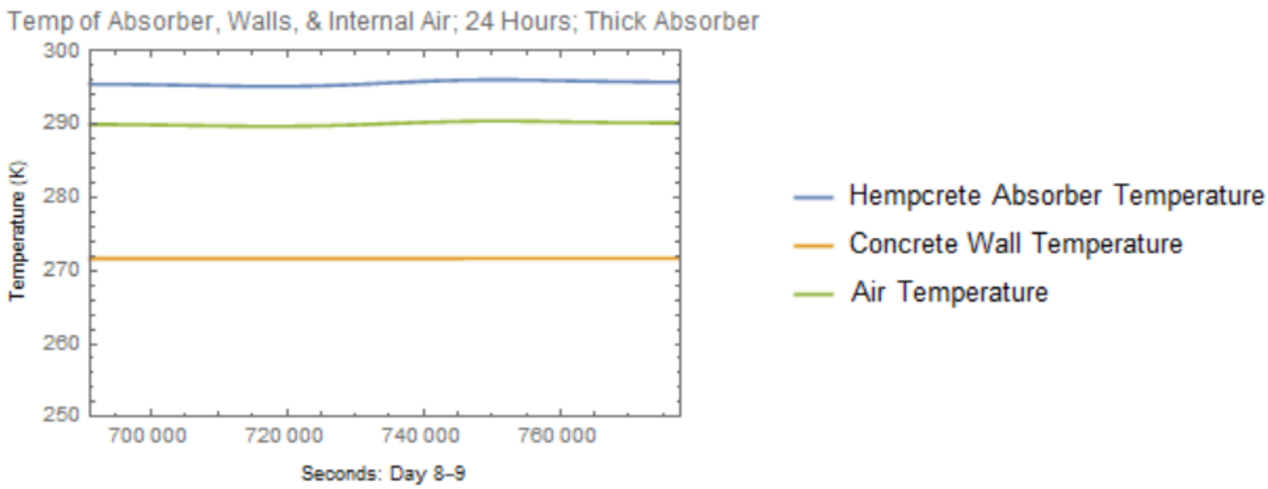

Figure 8: This figure shows the periodic plot of the temperature of the absorber, walls, and internal air over 24 hours with a thicker absorber of 20 meters

1.4 Optimization

Adjusting the values for insulation and absorber thickness to find the optimal pairing to match our desired temperature range was our main priority. Our absorber, which was made out of Hempcrete, was 2.2 meters thick. Our walls were made out of 3D printed concrete which was 0.15 meters thick. We chose these values after rigorous testing with our model.

We initially chose 0.2 meters for our absorber thickness and 0.15 for our insulation thickness. However, we saw that our house was remarkably cold and did not absorb enough heat for someone to live in the desired comfortable temperature range.

We noticed that when we increased the thickness of our absorber, the fluctuation of the air temperature would be less during the 24 hours since the absorber would be able to store more heat. When we decreased the thickness of our absorber, we saw that the temperature of the absorber and the inside air temperature were the same. This was not what we wanted since this would mean that our inside air temperature would be above our desired temperature range. With this logic, we knew that we wanted a thick absorber since we wanted the heat to last throughout the day.

For our walls, we noticed that really thin walls would mean that the heat would escape faster and the inside air temperature would be similar to the outside air. However, thicker walls would trap the heat and make the inside air temperature unbearable to live in. We knew that we wanted a reasonable wall thickness unlike our absorber thickness.

We chose 2.2 meters for our absorber thickness and 0.15 meters for our wall thickness after experimenting with our model. We wanted to try to limit our consumption of materials for our absorber and walls while also ensuring that our internal air temperature was in our desired range. For our next step, we set up our experiment to test the absorber thickness and confirm our suspicions that a thicker absorber would mean a more gradual increase in air temperature.

The predictive capability that we tested for $L_{insulation}$ and $L_{absorber}$ is accurate if we assume a perfect world and follow our assumptions. Major discrepancies include our assumptions and the model of the sun since the model for the sun's energy input is sinusoidal and becomes negative at some points in time. Our assumptions neglect heat loss through anywhere except the walls and a uniform internal temperature, walls, and absorber which is not an accurate representation of real life thermodynamics. Despite these discrepancies, we believe that our model can be applied to the general structure of our house.

1.5 Experiment

For our purposes, we decided to conduct two experiments to validate some aspects of our model.

For our first experiment, we decided to test the heat capacity of tile in order to get a better understanding of how other materials might react. Although our model is not composed of tile, we thought it had properties similar to that of our absorber (hempcrete).

Initially, we had an absorber that was 0.2 meters thick. After various iterations, we realized that our house would work best if our hempcrete was at least 2 meters thick. Therefore, we decided to change the thickness of the tile in our first experiment.

For the first portion of the experiment, we set up two different systems. One system was composed of a heating pad, a strip of tile and a thermistor. The other system was also composed of a heating pad and thermistor, but had two strips of tile. The set up can be seen in Figures 9 and 10.

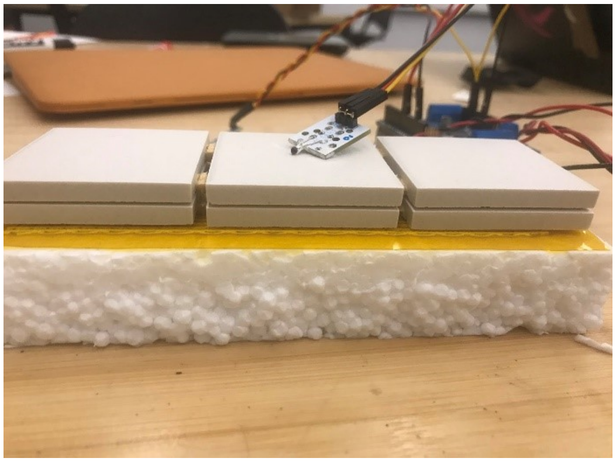

Figure 9: This figure shows our first system with only one tile layer

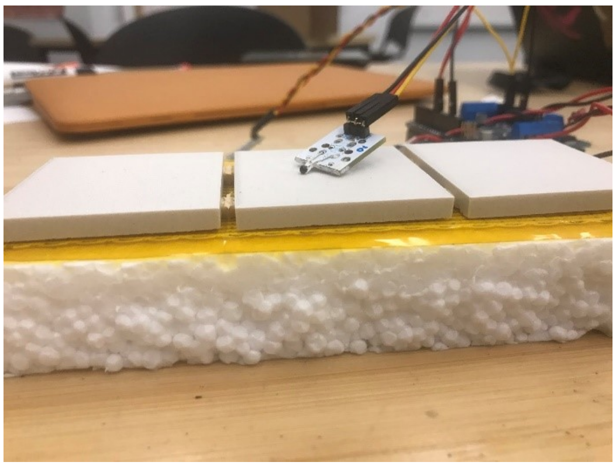

Figure 10: This figure shows our first system with a double tile layer

In this experiment, our goal was to determine how the thickness of a material affected the rate at which it would reach equilibrium within 24 minutes (hypothetically resembling 24 hours). As we had predicted, the thicker the material, the lower its final temperature and rate as seen in Figure 11.

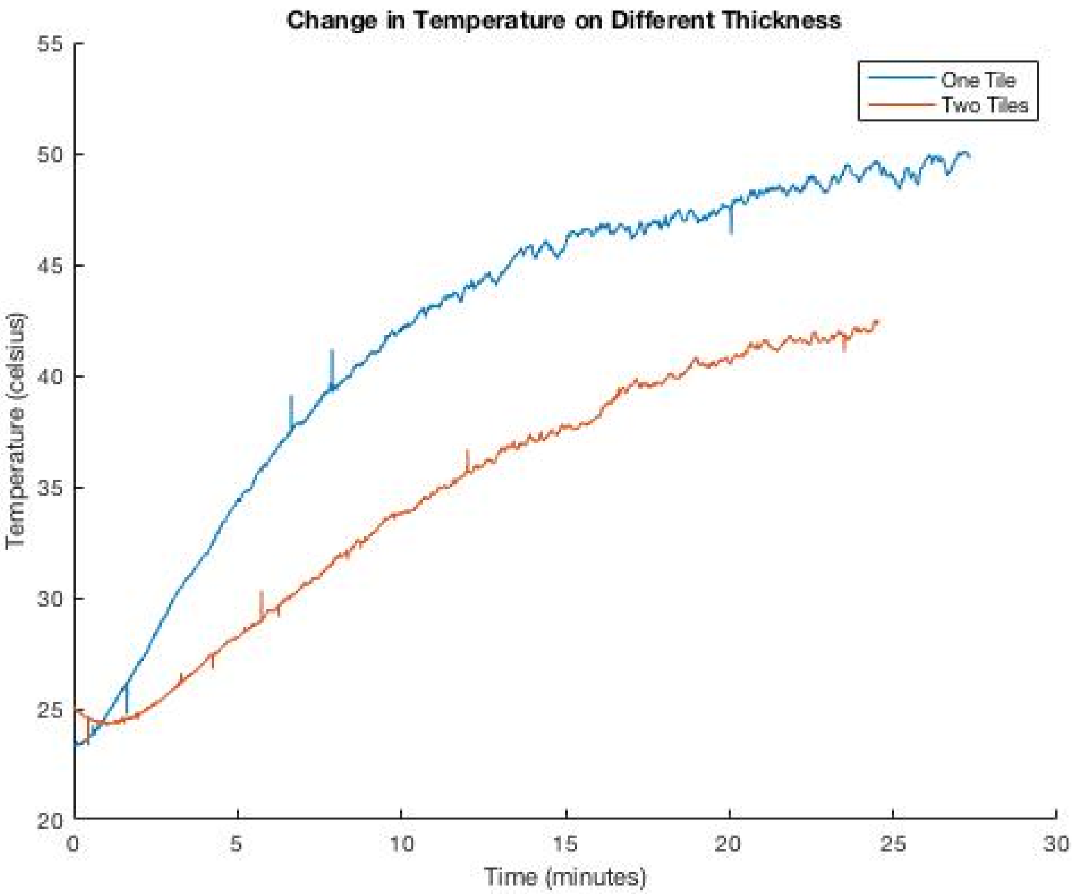

Figure 11: This figure shows the comparison of the change in temperature over a certain period of time between two different tile thickness.

For our second experiment, we wanted to simulate the difference in temperature between two floors by stacking two pieces of tile and using a thermistor in between to measure the temperature difference. Our setup is shown in Figure 12.

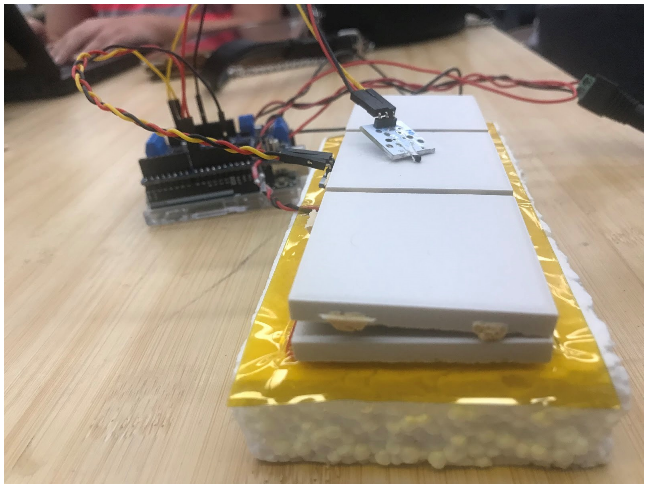

Figure 12: This figure shows the set up of a "two-story" house experiment with one thermistor on top of both tiles and another thermistor in between the tiles.

Some sources of error with the following set up were that we did not actually have a large space in between the two tiles to accurately represent a two-story house. Also, the thermistor created a wedge on one side that allowed air to flow through a certain area and did not heat the tile equally.

When starting this experiment, we predicted that the top would be warmer due to convection. Our hypothesis was incorrect according to this experiment. However, we believe it is mostly because of the way our experiment was arranged and the amount of time it was tested for. The final results can be seen in Figure 13. From this experiment, we decided that assuming uniform internal air temperature would be best for our model.

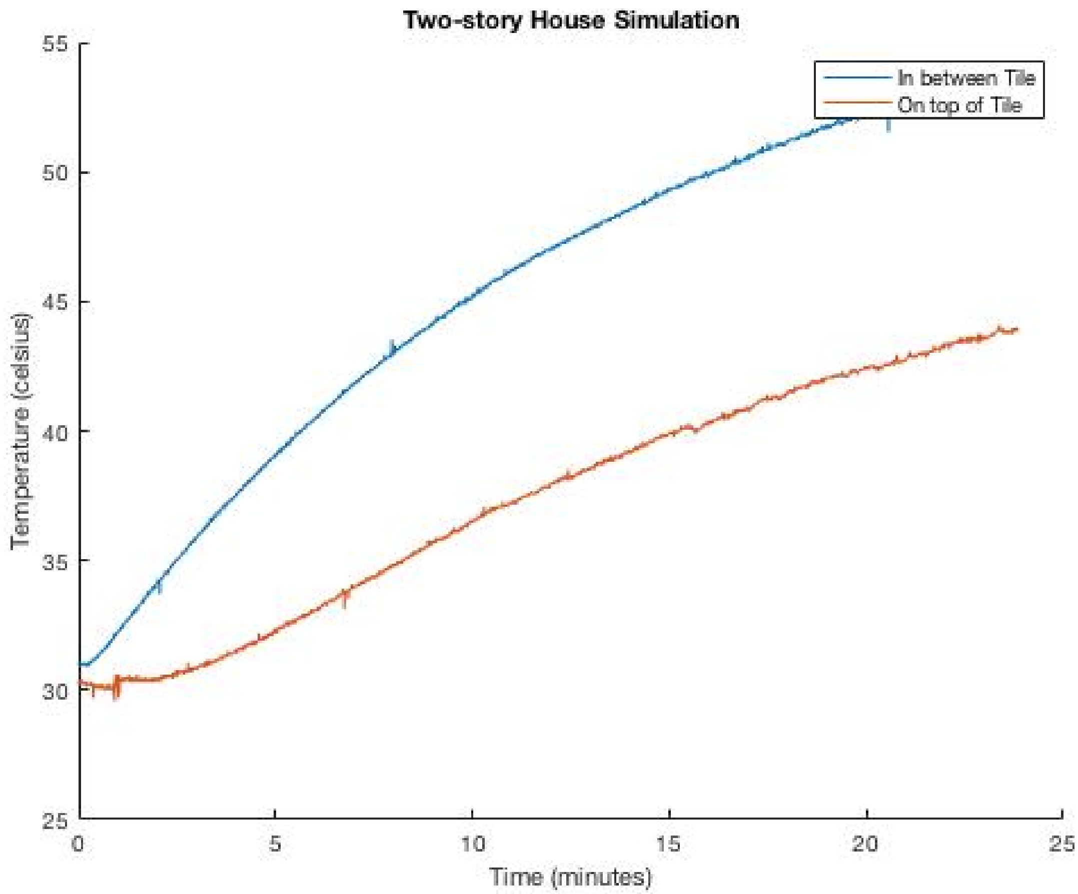

Figure 13: This figure shows the difference between the temperature of simulated "two-story" house in 24 minutes.

Although we did not get to test the materials we used in our model, we got a better understanding on how thickness and space between two objects affects its change in temperature.

2 Discussion

Our design appears to be practical and efficient based on the model results of the temperature over the course of a day. This means that the house design would likely be effective in sustaining humanoid residents within it without being subject to undesirable temperatures potentially caused by the environment or the house itself. The temperature range reported in our model are 19 degrees Celsius (approximately 66 degrees Fahrenheit) and 25 degrees Celsius (approximately 77 degrees Fahrenheit), which are generally perceived as comfortable temperatures. These results can be seen in Figures 3 and 4, as well as the temperatures for the walls and absorber for our customized values of 0.15 meters for $L_{insulation}$ and 2.2 meters $L_{absorber}$. Based on our experiment, it can be justified that the thickness of the absorber helps keep our model at a comfortable temperature. In Figure 11, it can be seen that the thicker the absorber, the slower the change in temperature. Our second experiment (Figure 13) was inconclusive, and we decided assuming a uniform internal air temperature would be best for our model. Overall, based on our results, the best factor of this home is that electricity and gasoline are not necessary to maintain typical dwelling temperatures (comfortable temperature range), which helps to set the human race on a path to a fossil-free civilization.

Future improvements to our home would be furniture, which could in turn affect the heat capture and distribution inside the home or building, as well as adding a second floor. Dimensions of materials (absorber and walls) may need to be varied based on the model since our model is only an approximation and is founded on various assumptions. A reevaluation of the model would be auspicious prior to permitting human residents to occupy a similar space. Beyond this concept of a tiny house, these materials can be further implemented into other building types (commercial, warehouse, etc) to diversify designs and materials for sustainable and thermally regulative buildings. Hempcrete has historically been used in bridge abutments and is a contruction material currently in rising popularity in the UK. Extrudable concrete has wide array of potential for use in not only buildings, but other structures, furnishings, or art pieces as technology further develops.

3 Code

The code written for this assignment can be found here.

4 Works Cited

"Boston's Carbon Emissions." Boston.gov, 2 Aug. 2019, https://www.boston.gov/departments/environment/bostons-carbon-emissions.

"Climate Change Evidence: How Do We Know?" NASA, NASA, 9 July 2019, https://climate.nasa.gov/evidence/.

\What is Hempcrete." American Lime Technology, 2019, http://www.americanlimetechnology.com/what-is-hempcrete/